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## THE VARIANCE OF RASCH ABILITY ESTIMATES FROM PARTIALLY-KNOWN ITEM PARAMETERS

Robert J. Mislevy Educational Testing Service



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Robert J. Mislevy, Principal Investigator



Educational Testing Service Princeton, New Jersey

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# The Variance of Rasch Ability Estimates from Partially-Known Item Parameters

Robert J. Mislevy

**Educational Testing Service** 

January, 1992

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# The Variance of Rasch Ability Estimates from Partially-Known Item Parameters

#### Abstract

A closed-form approximation is given for the variance of examinee proficiency estimates in the Rasch model for dichotomous items, under the condition that only estimates, rather than true values, of item difficulty parameters are available. The term that must be added to the usual response-sampling variance is inversely proportional to both the number of examinees in the item calibration sample and the length of the test. Illustrative numerical values suggest that the impact of uncertainty about Rasch item parameters on subsequent estimates of examinee proficiencies is less than has been observed in the 2- and 3-parameter logistic IRT models.

Key words: Cohen's approximation, Rasch model, variance components



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#### Introduction

It is common practice in applications of item response theory (IRT) to estimate item parameters from a calibration sample of examinees, then use these estimates as if they were known true parameters when estimating the proficiency parameters of these and subsequent examinees. Tsutakawa and his colleagues have pointed out that ignoring the uncertainty about item parameters leads to inferential errors, and offered a Taylor series approximation to incorporate this uncertainty into proficiency estimates and statements about their precision. Results for the two-parameter logistic IRT model appear in Tsutakawa and Soltys (1988) and for the three-parameter logistic model appear in Tsutakawa and Johnson (1990). (Also see Lewis, 1985, for an alternative approach to the problem, and Mislevy, Sheehan, & Wingersky, in press, for the application of these ideas to test equating.) Wright and Panchapekesan (1969) present without derivation a closed-form approximation for the variance of examinee proficiency estimates under the Rasch model for dichotomous items (Rasch, 1960/1980). Utilizing a series of computing approximations and simplifying assumptions, the present paper gives a somewhat simpler approximation. Neither numerical accuracy nor practical application is the point of this presentation—the aforementioned alternatives are preferable in this regard. Rather, by giving an approximation with fewer "moving parts," the objective is to aid intuition into the sources, the structures, and the consequences of uncertainty.

#### The Rasch Model

The Rasch model for dichotomous items gives the probability of a correct response to Item j from an examinee with proficiency  $\theta$  as

$$P_{j}(\theta) = \frac{\exp(\theta - b_{j})}{1 + \exp(\theta - b_{j})}, \qquad (1)$$

where  $b_j$  is the difficulty parameter of Item j. Letting  $x_j$  denote the response to Item j, 1 for correct and 0 for incorrect, the assumption of local independence gives the conditional probability of the response vector  $\mathbf{x} = (x_1, ..., x_L)$  on an L-item test as

$$P(\mathbf{x}|\boldsymbol{\theta}, \mathbf{b}) = \prod_{j=1}^{L} P_{j}(\boldsymbol{\theta})^{x_{j}} [1 - P_{j}(\boldsymbol{\theta})]^{1 - x_{j}},$$
(2)

where  $\mathbf{b} = (b_1, \dots, b_L)$ .

Given item difficulty parameters and a realization of  $\mathbf{x}$ , (2) can be interpreted as a likelihood function for  $\theta$ . Unless all the responses are correct or all are incorrect, it attains its maximum at a unique point, the maximum likelihood estimate (MLE)  $\hat{\theta}$ . The negative reciprocal of its second derivative at this point approximates the sampling variance of  $\hat{\theta}$ , or  $Var(\hat{\theta}|\theta,\mathbf{b})$ . Given response vectors from a calibration sample of N examinees, the product of expressions like (2) serves similarly as a basis for the estimation of  $\mathbf{b}$  and the sampling variance of the resulting estimate  $\hat{\mathbf{b}}$ .

### Cohen's Approximations

Cohen (1979) gives closed form approximations for  $\theta$ ,  $Var(\theta|\theta,b)$ ,  $\hat{b}$ , and  $Var(\hat{b}|b)$  for long tests when both  $\theta$  and b are normally distributed, in the samples of examinees and items respectively. Under these circumstances, item difficulty parameters can be approximated by rescaling the logits, or log-odds, of item percents-correct in the calibration sample, and examinees' proficiencies can be approximated by rescaling logits of their total scores by functions of item parameters. Adapting Wright's (1977) presentation, we begin with a calibration data set from which zero and perfect scoring examinees and items with all or no correct responses have been excised, then calculate the following quantities:

3

$$\overline{P}_{j}$$

the proportion of the calibration sample that answered Item j correctly, for j=1,...,L,

 $n_r$ 

the number of examinees with a total score of r, for r=1,...,L-1,

$$t_j = ln \left( \frac{1 - \overline{P}_j}{\overline{P}_j} \right)$$

the log ratio of wrong to right answers on Item j,

$$t_{\cdot} = \sum_{j=1}^{L} t_j / L$$

the mean of ti over L items,

$$T = \sum_{j=1}^{L} (t_{j}-t_{.})^{2}/(L-1)$$

the variance of ti over L items,

$$y_r = ln \left(\frac{r}{L-r}\right)$$

the log ratio of right to wrong answers for score r,

$$y_{\cdot} = \sum_{r=1}^{L-1} n_r y_r / N$$

the mean of y<sub>r</sub> over N examinees,

$$Y = \sum_{r=1}^{L-1} n_r (y_r - y_.)^2 / (N-1)$$

the variance of y<sub>r</sub> over N examinees, and

$$W = \left(\frac{1 + Y/2.89}{1 - TY/8.35}\right)^{1/2}$$

an expansion factor, in which 1.7 is used to match the logistic and normal distributions, and  $2.89=1.7^2$  and  $8.35=1.7^4$ .

Then, from Cohen (1979), item parameters can be approximated as follows:

$$\widehat{\mathbf{b}}_{\mathbf{j}} \approx \mathbf{W}(\mathbf{t}_{\mathbf{j}} - \mathbf{t}_{\mathbf{j}}) \tag{3}$$

and

$$Var(\widehat{b}_{j}|b_{j}) \approx \frac{1}{N} \left( \frac{W^{2}}{\overline{P}_{j}(1-\overline{P}_{j})} \right) \equiv \frac{C_{j}}{N} ,$$

where  $C_j=W^2/[\bar{P}_j(1-\bar{P}_j)]$ . The sampling covariances among the parameters of different items tend toward zero as L increases, as long as the items are not too dissimilar (Mislevy, 1981, shows that the error correlations among equivalent items are -1/(L-1)). We shall therefore approximate the L-by-L covariance matrix of sampling errors among item parameters as simply

$$\operatorname{Var}(\widehat{\mathbf{b}}|\mathbf{b}) \approx \operatorname{diag}(\frac{C_1}{N},...,\frac{C_L}{N})$$
 (4)

If item parameters are known, examinee proficiencies can be similarly approximated by rescaling logits of their total scores. Define three additional quantities:

$$b_{\cdot} = \sum_{j=1}^{L} b_j / L$$

the mean of bj over L items,

$$U = \sum_{j=1}^{L} (b_j - b_j)^2 / (L-1)$$

the variance of bi over L items, and

$$X = (1+U/2.89)^{1/2}$$

an expansion factor.

Then, for a total score of r,

$$\widehat{\theta} \approx b_{\cdot} + X \ln \left( \frac{r}{L-r} \right) = b_{\cdot} + X \ln \left( \frac{p_r}{1-p_r} \right), \tag{5}$$

where  $p_r \equiv r/L$ , and

$$\operatorname{Var}(\widehat{\boldsymbol{\theta}}|\boldsymbol{\theta}, \mathbf{b}) \approx X^2 \left(\frac{L}{r(L-r)}\right) \equiv \frac{1}{L} \left(\frac{X^2}{p_r(1-p_r)}\right).$$
 (6)

### Estimating Ability with Item Parameter Estimates

When **b** is known, (5) gives a proficiency estimate from **x**, and (6), evaluated with the true value of  $\theta$ , gives its sampling variance. What is the sampling variance of  $\widehat{\theta}$  when (5) is evaluated with  $\widehat{\mathbf{b}}$  rather than **b**? In this presentation, this amounts to using item parameter estimates rather than true parameter values to calculate **b** and **X**. The setup corresponds to a two-stage experiment:

- Stage 1: A sample of N examinees is drawn from a normal population. They respond to the L items in the test. Their responses are used to calculate  $\hat{\mathbf{b}}$  via (3).
- Stage 2: A new examinee is administered the test and the test score r is observed. A proficiency estimate is calculated using (5) with  $\hat{\mathbf{b}}$ . We can write this estimate as  $\hat{\theta}(\mathbf{r},\hat{\mathbf{b}})$  to emphasize that it is a function of two statistics—which, it will be noted, are independent, given  $\mathbf{b}$  and  $\theta$ .

The variance of  $\theta$  in this setting can be decomposed into two terms:

$$\operatorname{Var}(\widehat{\boldsymbol{\theta}}|\boldsymbol{\theta}) = \operatorname{E}_{\widehat{\boldsymbol{b}}} \left[ \operatorname{Var}(\widehat{\boldsymbol{\theta}}|\boldsymbol{\theta}, \widehat{\boldsymbol{b}}) \right] + \operatorname{Var}_{\widehat{\boldsymbol{b}}} \left[ \operatorname{E}(\widehat{\boldsymbol{\theta}}|\boldsymbol{\theta}, \widehat{\boldsymbol{b}}) \right]. \tag{7}$$

The first term on the right of (7) is the expectation of (6) as evaluated with  $\hat{\mathbf{b}}$ , over the distribution of  $\hat{\mathbf{b}}$  given  $\mathbf{b}$ . It depends on  $\mathbf{b}$  only through  $X^2$ , or 1+U/2.89, where U is the variance of the true item parameters. The expected variance of the item parameter estimates is the variance of the item parameters themselves plus the average estimation error variance,  $C_i/N=(\Sigma C_i/L)/N$ . Thus,

$$E_{\widehat{\mathbf{b}}} \left[ \operatorname{Var}(\widehat{\boldsymbol{\theta}} | \boldsymbol{\theta}, \widehat{\mathbf{b}}) \right] = E_{\widehat{\mathbf{b}}} \left[ \frac{1}{L} \left( \frac{\widehat{X^2}}{p_r (1 - p_r)} \right) \right]$$
$$= \frac{1}{L} \left( \frac{1 + (U + C/N)/2.89}{p_r (1 - p_r)} \right)$$

$$= \frac{1}{L} \left( \frac{X^2}{p_r (1-p_r)} \right) + \frac{C}{LN} \left( \frac{1}{2.89 p_r (1-p_r)} \right). \tag{8}$$

This is the sum of the error variance for  $\theta$  with known item parameters and a term depending on their error variance that is inversely proportional to both L and N.

The second term in (7) concerns the variation in  $\theta$  associated with the expected test score r for the true  $\theta$ , as induced by variation in  $\hat{\mathbf{b}}$ . This term can be approximated by the delta method as follows:

$$\begin{aligned} & \text{Var}_{\widehat{\mathbf{b}}} \Big[ F(\widehat{\boldsymbol{\theta}} | \boldsymbol{\theta}, \widehat{\mathbf{b}}) \Big] \approx \frac{\partial \widehat{\boldsymbol{\theta}}}{\partial \widehat{\mathbf{b}}'} \, \text{Var}(\widehat{\mathbf{b}} | \mathbf{b}) \, \frac{\partial \widehat{\boldsymbol{\theta}}}{\partial \widehat{\mathbf{b}}} \\ & \approx \sum_{j=1}^{L} \left[ \left\{ \frac{\partial \widehat{\boldsymbol{\theta}}}{\partial \widehat{\mathbf{b}}_{j}} \right\}^{2} \, \text{Var}(\widehat{\mathbf{b}}_{j} | \mathbf{b}_{j}) \right] \\ & = \sum_{j=1}^{L} \left[ \left\{ \frac{\partial}{\partial \widehat{\mathbf{b}}_{j}} \left[ \widehat{\mathbf{b}}_{.} + \widehat{\mathbf{X}} \, \ln \left( \frac{\mathbf{p}_{r}}{1 - \mathbf{p}_{r}} \right) \right] \right\}^{2} \, \text{Var}(\widehat{\mathbf{b}}_{j} | \mathbf{b}_{j}) \right] \\ & \approx \sum_{j=1}^{L} \left[ \frac{1}{L} + \frac{(\mathbf{b}_{j} - \mathbf{b}_{.}) \, \ln \left( \frac{\mathbf{p}_{r}}{1 - \mathbf{p}_{r}} \right)}{2.89 \, LX} \right]^{2} \frac{C_{j}}{N} \, . \end{aligned}$$

Approximating all C<sub>j</sub>s by their average, say C<sub>.</sub>, leads to the following simplification:

$$\operatorname{Var}_{\widehat{\mathbf{b}}}\left[\widehat{E(\theta|\theta,\widehat{\mathbf{b}})}\right] \approx \frac{C}{LN}\left[1 + \frac{U\left\{\ln\left(\frac{p_{r}}{1-p_{r}}\right)\right\}^{2}}{8.35 X^{2}}\right]. \tag{9}$$

Substituting (8) and (9) back into (7) yields the final approximation:

$$Var(\widehat{\theta}|\theta) \approx \frac{1}{L} \left( \frac{X^2}{p_r(1-p_r)} \right) + \frac{C}{LN} \left( 1 + \frac{1}{2.89 p_r(1-p_r)} + \frac{U \left( ln \left( \frac{p_r}{1-p_r} \right) \right)^2}{8.35 X^2} \right)$$

$$\approx \operatorname{Var}(\widehat{\boldsymbol{\theta}}|\boldsymbol{\theta}, \mathbf{b}) + \frac{C}{LN} \left( 1 + \frac{1}{2.89 \, p_r (1 - p_r)} + \frac{U \left( \ln \left( \frac{p_r}{1 - p_r} \right) \right)^2}{8.35 \, X^2} \right). \tag{10}$$

Thus the sampling variance of  $\theta$  with item parameter estimates can be approximated as the sum of its variance with true item parameters and a correction term. The following observations can be made about the correction term:

- 1. It is always positive.
- 2. It is inversely proportional to test length (L).
- 3. It is inversely proportional to the size of the examinee calibration sample (N).
- 4. It is directly proportional to C. Since

$$C_j = \frac{\left(\frac{1+Y/2.89}{1-TY/8.35}\right)}{\overline{P}_j(1-\overline{P}_j)},$$

where Y is the variance of the logits of item percents-correct and  $\bar{P}_j$  is the percent-correct for Item j in the calibration sample, the impact of the uncertainty about item parameters is greater when...

- a. the items are more dispersed, so that Y departs from zero, or
- b. items are farther from 50-percent correct, so that  $\bar{P}_j(1-\bar{P}_j)$  terms fall farther from the maximum value of .25.
- Values of θ for which expected proportions of correct response are near 50-percent are less affected by uncertainty about item parameters than more extreme values of θ. This follows from the second and third terms inside the brackets in (10): As p<sub>r</sub> approaches .5, the second term approaches its minimum value of 4/2.89, and, because the logit of .5 is zero, the third term approaches zero.

#### Illustrative Numerical Values

To provide a feel for the relative contributions of sources of uncertainty this section evaluates the preceding formulae with some illustrative values. Standard normal distributions are assumed for  $\theta$  and b in the item calibration sample, implying the following values for true parameter values:

t. = y. = b. = 0  

$$T = Y = .743$$
  
 $X = W = 1.160$   
 $U = 1$ .

Equation (10) is used to approximate  $\theta$  sampling variances, for  $\theta = 0$  and 2, for test lengths of 10, 20, 40, 80, and 160 items, after item calibration with examinee samples of 50, 100, 250, 500, 1000, 2500, and 10000. For C., we employ a representative value, namely that for an item one standard deviation from the mean. For such an item,  $\tilde{P}$  is about .7, in which case

$$C_{x} = W^{2}/(.7 \times .3) = 6.40$$
.

The values of  $p_r$  that correspond to  $\theta = 0$  and 2 are .50 and .85.

Table 1 gives results for  $\theta = 0$ , in terms of approximate sampling variances and proportional increases in sampling variance over those based on known item parameters. The proportional increases in standard errors are the square roots of the values in the rightmost column. Table 2 gives similar results for  $\theta = 2$ .

The most striking feature of these results is how small the corrections are. Even with a calibration sample of only 50 examinees, estimation error variances for subsequent  $\theta$  estimates increase by only about 5-percent. This contrasts with the increases of up to 30-percent Tsutakawa and Soltys (1988) observed in posterior variances for  $\theta$  with a

calibration sample of 100 under the 2-parameter logistic model. Tsutakawa and Johnson's (1990) results for the 3-parameter logistic were even more extreme, with increases in posterior variances more than doubling at higher levels of proficiency with a calibration sample of 400.

#### Conclusion

Cohen's (1979) closed-form approximations for the parameters in the Rasch model support a closed-form approximation for the variance of these examinee proficiency estimates when they are calculated with item parameter *estimates* rather than true item parameter values. The approximation is the sum of the sampling variance with known item parameters and a correction term. The correction term is inversely proportional to both test length and the size of the examinee item-calibration sample. Illustrative numerical values suggests the additional variance is quite small, yielding increases in standard errors of less than 5-percent even with calibration samples of only 50 examinees.

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Table 1
Approximate Error Variances at θ=0

			L			
N	10	20	40	80	160	$\frac{\widehat{\operatorname{Var}(\boldsymbol{\theta} \boldsymbol{\theta},\mathbf{b})}}{\widehat{\operatorname{Var}(\boldsymbol{\theta} \boldsymbol{\theta},\boldsymbol{\hat{b}})}}$
50	0.5689	0.2845	0.1422	0.0711	0.0356	1.0567
100	0.5537	0.2768	0.1384	0.0692	0.0346	1.0283
250	0.5445	0.2723	0.1361	0.0681	0.0340	1.0113
500	0.5415	0.2707	0.1354	0.0677	0.0338	1.0057
1000	0.5399	0.2700	0.1350	0.0675	0.0337	1.0028
2500	0.5390	0.2695	0.1348	0.0674	0.0337	1.0011
10000	0.5386	0.2693	0.1346	0.0673	0.0337	1.0003
<u> </u>	0.5384	0.2692	0.1346	0.0673	0.0337	1.0000

Table 2  $\label{eq:table 2} Approximate Error Variances at $\theta$=2$ 

			L			
N	10	20	40	80	160	$\frac{\widehat{\mathrm{Var}(\boldsymbol{\hat{\theta}} \boldsymbol{\theta},\mathbf{b})}}{\widehat{\mathrm{Var}(\boldsymbol{\hat{\theta}} \boldsymbol{\theta},\widehat{\mathbf{b}})}}$
50	1.1066	0.5533	0.2767	0.1383	0.0692	1.0483
100	1.0812	0.5406	0.2703	0.1351	0.0676	1.0241
250	1.0659	0.5329	0.2665	0.1332	0.0666	1.0097
500	1.0608	0.5304	0.2652	0.1326	0.0663	1.0048
1000	1.0582	0.5291	0.2646	0.1323	0.0661	1.0024
2500	1.0567	0.5284	0.2642	0.1321	0.0660	1.0010
10000	1.0559	0.5280	0.2640	0.1320	0.0660	1.0002
<b>00</b>	1.0557	0.5278	0.2639	0.1320	0.0660	1.0000

Dr. Terry Ackerman Educational Psychology 260C Education Bidg, University of Illinois Champaign, IL 61801

Dr. Terry Allard Code 1142CS Office of Naval Research 800 N. Quincy St. Arlington, VA 22217-5000

Dr. Nancy Allen Educational Testing Service Princeton, NJ 08541

Dr. Gregory Anrig Educational Testing Service Princeton, NJ 08541

Dr. Phipps Arabie Graduate School of Management Rutgers University 92 New Street Newarts, NJ 07102-1895

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Dr. Menucha Birenbaum Educational Testing Service Princeton, NJ 08541

Dr. Bruce Blosses Defense Manpower Data Center 99 Pacific St. Suite 155A Monterey, CA 93943-3231

Dr. Gwyneth Boodoo Educational Testing Service Princeton, NJ 08541

Dr. Richard L. Branch HQ. USMEPCOM/MEPCT 2500 Green Bay Road North Chicago, IL 40064

Dr. Robert Brennan American College Testing Programs P. O. Box 168 Iowa City, IA 52243

Dr. Devid V. Budescu Department of Psychology University of Haifa Mount Carmet, Haifa 31999 ISRAEL

Dr. Gregory Candell CTB/MacMillan/McGraw-Hill 2509 Garden Road Monterey, CA 93940

Dr. Paul R. Chatelier Perceptronics 1911 North Ft. Myer Dr. Suite 1809 Arlington, VA 22209 Dr. Susan Chipman Cognitive Science Program Office of Navat Research 800 North Quincy St. Arlington, VA 22217-5000

Dr. Raymond E. Christal UES LAMP Science Advisor AL/HRMIL Brooks AFB, TX 78235

Dr. Norman Cliff Department of Psychology Univ. of So. California Los Angeles, CA 90089-1061

Director Life Sciences, Code 1142 Office of Naval Research Arlington, VA 22217-5000

Commanding Officer Naval Research Laboratory Code 4827 Washington, DC 20375-5000

Dr. John M. Cornwell
Department of Psychology
I/O Psychology Program
Tulane University
New Orleans, LA 70118

Dr. William Crano Department of Psychology Texas A&M University College Station, TX 77843

Dr. Linda Curran Defense Manpower Data Center Suite 400 1600 Wilson Blvd Rosslyn, VA 22209

Dr. Timothy Davey American College Testing Program P.O. Box 168 Iowa City, IA 52243

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Dr. Fritz Drasgow University of Illinois Department of Psychology 603 E. Daniel St. Champaign, IL 61820

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Dr. Susan Embretson University of Kansas Psychology Department 426 Fraser Lawrence, KS 66045

Dr. George Engelbard, Jr. Division of Educational Studies Emory University 210 Fishburne Bidg, Atlants, GA 30322

ERIC Facility-Acquisitions 2440 Research Blvd., Suite 550 Rockville, MD 20850-3238

Dr. Marshall J. Farr Farr-Sight Co. 2520 North Vernon Street Arlington, VA 22207

Dr. Leonard Feldt Lindquist Center for Measurement University of Iowa Iowa City, IA 52242

Dr. Richard L. Ferguson American College Testing P.O. Box 168 Iowa City, IA 52243

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Chair, Department of Computer Science George Mason University Fairfax, VA 22030

Dr. Robert D. Gibbons University of Illinois at Chicago NPI 909A, M/C 913 912 South Wood Street Chicago, IL, 60612

Dr. Janice Gifford University of Massachusetts School of Education Amberst, MA 01003

Dr. Robert Glaser Learning Research & Development Center University of Pittsburgh 3939 O'Hara Street Pittsburgh, PA 15260

Dr. Susan R. Goldman Peabody College, Box 45 Vanderbit University Nashville, TN 37203

Dr. Timothy Goldsmith Department of Psychology University of New Mexico Albuquerque, NM 87131

Ł

Dr. Joseph McLachlan Navy Personnet Research and Development Center Code 14 San Diego, CA 92152-6800

Alan Mead t/o Dr. Michael Levine Educational Psychology 210 Education Bldg. University of Illinois Champsign, IL 61801

Dr. Timothy Miller ACT P. O. Box 168 Iowa City, IA 52243

Dr. Robert Mislevy Educational Testing Service Princeton, NJ 08541

Dr. Ivo Molenar
Faculteit Sociale Wetenschappen
Rijksuniversiteit Groningen
Grote Kruisstrast 2/1
9712 TS Groningen
The NETHERLANDS

Dr. E. Muraki Educational Testing Service Rosedale Road Princeton, NJ 08541

Dr. Ratna Nandakumar Educational Studies Willard Hall, Room 213E University of Delaware Newark, DE 19716

Academic Progs. & Research Branch Naval Technical Training Command Code N-62 NAS Memphia (75) Millington, TN 30854

Dr. W. Alan Nioewander University of Oklahoma Department of Psychology Norman, OK 73071

Head, Personnel Systems Department NPRDC (Code 12) San Diego, CA 92152-6800

Director Training Systems Department NPRDC (Code 14) San Diego, CA 92152-6800

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Special Assistant for Research
Menagement
Chief of Naval Personnel (PERS-OUT)
Department of the Navy
Washington, DC 20350-2008

Dr. Judith Orsents Mail Stop 239-1 NASA Arnes Research Center Moffett Field, CA 94035 Dr. Peter J. Pashley Educational Testing Service Rosedale Road Princeton, NJ 08541

Wayne M. Patience American Council on Education GED Testing Service, Suite 20 One Dupont Circle, NW Washington, DC 20036

Dept. of Administrative Science Code 54 Naval Postgraduate School Monterey, CA 93943-5026

Dr. Peter Pirolli School of Education University of California Berkeley, CA 94720

Dr. Mark D. Reckase ACT P. O. Box 168 lows City, IA 52243

Mr. Steve Reise Department of Psychology University of California Riverside, CA 92521

Mr. Louis Roussos University of Illinois Department of Statistics 101 Illini Hall 725 South Wright St. Champaign, IL 61820

Dr. Donald Rubin Statistics Department Science Center, Room 608 1 Oxford Street Harvard University Cambridge, MA 02138

Dr. Fumiko Samejima Department of Psychology University of Tennessee 3108 Austin Peny Bldg, Knowille, TN 37966-0900

Dr. Mary Schratz 4100 Parkside Carlsbad, CA 92008

Mr. Robert Semmes N218 Elliott Hall Department of Psychology University of Minnesota Minneapolis, MN 55455-0344

Dr. Valeric L. Shalin Department of Industrial Engineering State University of New York 342 Lawrence D. Bett Hall Buffalo, NY 14260

Mr. Richard J. Shavelson Graduate School of Education University of California Senta Barbara, CA 93106

Ms. Kathleen Sheeban Educational Testing Service Princeton, NJ 08541

Dr. Kazuo Shigemasu 7-9-24 Kugenuma-Kaigan Fujimwa 251 JAPAN

Dr. Randall Shumaker Naval Research Laboratory Code 5500 4555 Overlook Avenue, S.W. Washington, DC 20375-5000 Dr. Judy Spray ACT P.O. Box 168 Iowa City, IA \$2243

Dr. Martha Stocking Educational Testing Service Princeton, NJ 08541

Dr. William Stout University of Illinois Department of Statistics 101 Illini Hall 725 South Wright St. Champaign, IL 61820

Dr. Kikumi Tatauoka Educational Testing Service Mail Stop 03-T Princeton, NJ 08541

Dr. David Thissen
Psychometric Laboratory
CB# 3270, Davie Hall
University of North Carolina
Chapel Hill, NC 27599-3270

Mr. Thomas J. Thomas Federal Express Corporation Human Resource Development 3035 Director Row, Suite 501 Memphis, TN 38131

Mr. Gary Thomasson University of Illinois Educational Psychology Champaign, IL, 61820

Dr. Howard Wainer Educational Testing Service Princeton, NJ 08541

Elizabeth Wald Office of Navat Technology Code 227 800 North Quincy Street Arlington, VA 22217-5000

Dr. Michael T. Waller University of Wisconsin-Milwaukee Educational Psychology Dept. Box 413 Milwaukee, WI 53201

Dr. Ming-Mei Wang Educational Testing Service Mail Stop 63-T Princeton, NJ 66541

Dr. Thomas A. Warm FAA Academy P.O. Box 25082 Oklahoma City, OK 73125

Dr. David J. Weiss N660 Elliott Hall University of Minnesota 75 E. River Road Minneapolis, MN 55455-0344

Dr. Douglas Wetzel Code 15 Navy Personnel R&D Center San Diego, CA 92152-6800

German Military Representative Personalstamment Koelner Str. 262 D-5000 Koeln 90 WEST GERMANY Dr. Sherrie Gott AFHRL/MOMJ Brooks AFB, TX 78235-5601

Dr. Bert Green Johns Hopkins University Department of Psychology Charles & 34th Street Baltimore, MD 21218

Prof. Edward Haertel School of Education Stanford University Stanford, CA 94305-3096

Dr. Rorsald K. Hambleton University of Massachusetta Laboratory of Psychometric and Evaluative Research Hills South, Room 152 Amherst, MA 01003

Dr. Delwyn Harnisch University of Illinois 51 Gerty Drive Champaign, IL 61820

Dr. Patrick R. Harrison Computer Science Department U.S. Naval Academy Annapolis, MD 21402-5002

Ms. Rebecca Hetter Navy Personnel R&D Center Code 13 San Diego, CA 92152-6800

Dr. Thomas M. Hirsch ACT P. O. Box 168 Iowa City, IA 52243

Dr. Paul W. Holland Educational Testing Service, 21-T Rosedale Road Princeton, NJ 08541

Prof. Lutz F. Hornke Institut für Psychologie RWTH Aachen Jaegerstrasse 17/19 D-5100 Aachen WEST GERMANY

Ms. Julia S. Hough Cambridge University Press 40 West 20th Street New York, NY 10011

Dr. William Howell
Chief Scientist
AFHRL/CA
Brooks AFB, TX 78235-5601

Dr. Huynb Huynb Coffege of Education Univ. of South Carolina Columbia, SC 29208

Dr. Martin J. Ippel
Center for the Study of
Education and Instruction
Leiden University
P. O. Box 9555
2300 RB Leiden
THE NETHERLANDS

Dr. Robert Jannarone Elec. and Computer Eng. Dept. University of South Carolina Columbia, SC 29208 Dr. Kumar Joag-dev University of Illinois Department of Statistics 101 Illini Hall 725 South Wright Street Champaign, IL 61820

Professor Douglas H. Jones Graduate School of Management Rutgers, The State University of New Jersey Newark, NJ 07102

Dr. Brian Junker Carnegie-Mellon University Department of Statistics Pittsburgh, PA 15213

Dr. Marcel Just Carnegie-Mellon University Department of Psychology Schenley Park Pittsburgh, PA 15213

Dr. J. L. Kaiwi Code 442/JK Naval Ocean Systems Center San Diego, CA 92152-5000

Dr. Michael Kaplan
Office of Basic Research
U.S. Army Research Institute
5001 Eisenbower Avenue
Alexandria, VA 22333-5600

Dr. Jeremy Kilpatrick
Department of
Mathematics Education
105 Aderbold Hall
University of Georgia
Athens, GA 30602

Ms. Hae-Rim Kim University of Illinois Department of Statistics 101 Illini Hall 725 South Wright St. Champaign, IL 61820

Dr. Jwa-keun Kim
Department of Psychology
Middle Tennessee State
University
Murfreesboro, TN 37132

Dr. Sung-Hoon Kim KEDI 92-6 Umyeon-Dong Seocho-Gu Seoul SOUTH KOREA

Dr. G. Gage Kingsbury Portland Public Schools Research and Evaluation Department 50 North Dison Street P. O. Box 3107 Portland, OR 97209-3107

Dr. William Koch Box 7246, Mess. and Evel. Ctr. University of Texas-Austin Austin, TX 78703

Dr. James Krastz Computer-based Education Research Laboratory University of Illinois Urbana, IL 61801

Dr. Patrick Kyllonen AFHRL/MOEL Brooks AFB, TX 78235

Ms. Carolyn Laney 1515 Spencerville Rod Spencerville, MD 20868 Richard Lanterman Commandant (G-PWP) US Coast Guard 2100 Second St., SW Washington, DC 20593-0001

Dr. Michael Levine Educational Psychology 210 Education Bidg. 1310 South South Street University of IL at Urbana-Champeign Champeign, IL 6120-6990

Dr. Charles Lewis Educational Testing Service Princeton, NJ 08541-0001

Mr. Hsin-hung Li University of Illinois Department of Statistics 101 Illini Hall 725 South Wright St. Champaign, IL 61820

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Dr. Marcia C. Linn Graduate School of Education, EMST Tolman Hall University of California Berkeley, CA 94720

Dr. Robert L. Linn Campus Box 249 University of Colorado Boulder, CO 80309-0249

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Dr. Richard Luecht ACT P. O. Box 168 Iowa City, IA 52243

Dr. George B. Macready Department of Measurement Statistics & Evaluation College of Education University of Maryland College Park, MD 20742

Dr. Evans Mandes George Mason University 4400 University Drive Fairfax, VA 22030

Dr. Paul Mayberry Center for Naval Analysis 4401 Ford Avenue P.O. Box 16268 Alexandria, VA 22302-0268

Dr. James R. McBride HumRRO 6430 Elmburst Drive San Diego, CA 92120

Mr. Christopher McCusker University of Illinois Department of Psychology 603 E. Daniel St. Champaign, IL 61820

Dr. Robert McKinley Educational Testing Service Princeton, NJ 98541 Dr. David Wiley School of Education and Social Policy Northwestern University Evanston, IL 60208

Dr. Bruce Williams
Department of Educational
Psychology
University of Illinois
Urbana, IL 61801

Dr. Mark Wilson School of Education University of California Berkeley, CA 94720

Dr. Eugene Winograd Department of Psychology Emory University Atlanta, GA 30322

Dr. Martin F. Wiskoff PERSEREC 99 Pacific St., Suite 4556 Monterey, CA 93940

Mr. John H. Wolfe Navy Personnel R&D Center San Diego, CA 92152-6800

Dr. Kentaro Yamamoto 03-07 Educational Testing Service Rosedale Road Princeton, NJ 08541

Ms. Duanti Yan Educational Testing Service Princeton, NJ 08541

Dr. Wendy Yen CTB/McGraw Hill Del Monte Research Park Montercy, CA 93940

Dr. Joseph L. Young National Science Foundation Room 320 1800 G Street, N.W. Washington, DC 20550